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Treating Top Differently from Charm and Up*

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TREATING TOP DIFFERENTLY FROM CHARM AND UP

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Abstract

It now appears phenomenologically that the third family may be essentially different from the first two. Particularly the high value of the top quark mass suggests a special role. In the standard model all three families are treated similarly [becoming exactly the same at asymptotically high energies] so I need to extend the model to accommodate the goal of a really different third family. In this article I describe not one but two such viable extensions, quite different one from another. The first is the 331 model which predicts dileptonic gauge bosons. In the second, using as a flavor symmetry a finite nonabelian dicyclic Q_{2N} group, I show how to derive quark mass matrices with two arrangements of symmetric texture zeros which are phenomenologically viable. Three other such acceptable textures in the recent literature are unattainable in this approach and hence disfavored. I assume massive vector-like fermions and Higgs singlets transforming as judiciously-chosen Q_{2N} doublets and use the tree-level mass generation mechanism of Froggatt and Nielsen.

In this presentation, I shall address two extensions of the standard model in which the third family is dealt with asymmetrically with respect to the lightest two. First, I shall describe the 331 Model and elaborate on its predictions including the dilepton, the neutrino masses and tests in hadronic and leptonic colliders. Second, after a review of finite nonabelian groups of order ≤ 31 and the question of their gauging and of their chiral anomalies, model building in that direction is discussed culminating in models based on the dicyclic groups Q_{2N} .

331 Model

Family symmetry is usually taken to mean a horizontal symmetry, either global or gauged, under which the three families transform under some non-trivial representation. The family symmetry is *broken* in order to avoid unobserved mass degeneracies. In this meaning of family symmetries, there is usually no explanation of *why* there are three families which are the input. Rather the hope is that the postulated family symmetry may explain the observed hierarchies:

$$m_u \ll m_c \ll m_t \tag{1}$$

$$m_d \ll m_c \ll m_b \tag{2}$$

$$\theta_{13} < \theta_{23} < \theta_{12} \tag{3}$$

In the present model, the aim of family symmetry is indeed to attempt to address such hierarchies *and* to explain why there are three families. This may be a necessary first step to understanding hierarchies?

To introduce the 331 Model[1] the following are motivating factors:

i) Consistency of a gauge theory (unitarity, renormalizability) requires anomaly cancellation. This requirement almost alone is able to fix all electric charges and other quantum numbers within one family of the standard model. This accounts for charge quantization, *e.g.* the neutrality of the hydrogen atom, without the need for a GUT.

ii) This does not explain why $N_f > 1$ for the number of families but is sufficiently impressive to suggest that $N_f = 3$ may be explicable by anomaly cancellation in an extension of the standard model. This requires that each extended family have non-vanishing anomaly and that the three families are not all treated similarly.

iii) A striking feature of the mass spectrum in the SM is the top mass suggesting that the 3rd. family be treated differently and that the anomaly cancellation be proportional to: $+1 +1 -2 = 0$.

iv) There is a " -2 " lurking in the SM in the ratio of the quark electric charges!

v) The electroweak gauge group extension from $SU(2)$ to $SU(3)$ will add five gauge bosons. The adjoint of $SU(3)$ breaks into $8 = 3 + (2 + 2) + 1$ under $SU(2)$. The 1 is a Z' and the two doublets are readily identifiable from the leptonic triplet or antitriplet (e^-, ν_e, e^+) as *dilepton* gauge bosons (Y^{--}, Y^-) with $L = 2$ and (Y^{++}, Y^+) with $L = -2$. Such dileptons appeared first in stable-proton GUTs but there the fermions were non-chiral and one needed to invoke mirror fermions; this is precisely what is avoided in the 331 Model. But it is true that the $SU(3)$ of the 331 Model has the same couplings to the *leptons* as that of the leptonic $SU(3)_l$ subgroup of $SU(15)$ which breaks to $SU(12)_q \times SU(3)_l$.

Now I am ready to introduce the 331 Model in its technical details: the gauge group of the standard model is extended to $SU(3) \times SU(3) \times U(1)$ where the electroweak $SU(3)$ contains the standard $SU(2)$ and the weak hypercharge is a mixture of λ_8 with the $U(1)$. The leptons are in the antitriplet $(e^-, \nu_e, e^+)_L$ and similarly for the μ and τ .

These antitriplets have $X = 0$ where X is the new $U(1)$ charge. This can be checked by noting that the X value is the electric charge of the central member of the triplet or antitriplet.

For the first family of quarks I use the triplet $(u, d, D)_L$ with $X = -1/3$ and the right-handed counterparts in singlets. Similarly, the second family of quarks is treated. For the third family of quarks, on the other hand, I use the *antitriplet* $(T, t, b)_L$ with $X = +2/3$. The new exotic quarks D, S, and T have charges $-4/3$, $-4/3$ and $+5/3$ respectively.

It is instructive to see how this combination successfully cancels all chiral anomalies:

The purely color anomaly $(3_L)^3$ cancels because QCD is vector-like.

The anomaly $(3_L)^3$ is non-trivial. Taking, for the moment, arbitrary numbers N_c of colors and N_l of light neutrinos I find this anomaly cancels only if $N_c = N_l = 3$.

The remaining anomalies $(3_c)^2 X$, $(3_L)^2 X$, X^3 and $X(T_{\mu\nu}^2)$ also cancel.

Each family separately has non-zero anomaly for X^3 , $(3_L)^2 X$ and $(3_L)^3$; in each case, the anomalies cancel proportionally to $+1 + 1 - 2$ between the families.

To break the symmetry I need several Higgs multiplets. A triplet Φ with $X = +1$ and $\text{VEV} < \Phi > = (0, 0, U)$ breaks 331 to the standard 321 group, and gives masses to D, S, and T as well as to the gauge bosons Y and Z' . The scale U sets the range of the new physics and I shall discuss more about its possible value.

The electroweak breaking requires two further triplets ϕ and ϕ' with $X = 0$ and $X = -1$ respectively. Their VEVs give masses to d, s, t and to u, c, b respectively. The first VEV also gives a contribution of an antisymmetric-in-family type to the charged

leptons. To complete a satisfactory lepton mass matrix necessitates adding a sextet with $X = 0$.

What can the scale U be? It turns out that there is not only the lower bound expected from the constraint of the precision electroweak data, but also an upper bound coming from a group theoretical constraint within the theory itself.

The lower bound on U from $Z - Z'$ mixing can be derived from the diagonalization of the mass matrix and leads to $M(Z') \geq 300\text{GeV}$. The limit from FCNC (the Glashow - Weinberg rule is violated) gives a similar bound; here the suppression is helped by ubiquitous $(1 - 4\sin^2\theta)$ factors.

In these considerations, particularly with regard to FCNC, the special role played by the third family is crucial; if either of the first two families is the one treated asymmetrically the FCNC disagree with experiment.

The upper bound on U arises because the embedding of the standard 321 group in 331 requires that $\sin^2\theta \leq 1/4$. When $\sin^2\theta = 1/4$, the $SU(2) \times U(1)$ group embeds entirely in $SU(3)$, and the coupling of the X charge in principle diverges. Because the phenomenological value is close to $1/4$ - actually $\sin^2\theta(M_Z) = 0.233$ - the scale U must be less than about 3TeV after scaling $\sin^2\theta(\mu)$ by the renormalization group. Putting some reasonable upper bound on the X coupling leads to an upper bound on the dilepton mass, for this 331 Model, of about 800GeV [Here I have allowed one further Higgs multiplet - an octet].

A very useful experiment for limiting the dilepton mass from below is polarized muon decay. With the coupling parametrized as $V - \xi A$ where ξ is a Michel parameter, the present limit on ξ is $1 \geq \xi \geq 0.997$ coming from about 10^8 examples of the decay. This leads to a lower bound $M(Y) \geq 300\text{GeV}$.

Since

$$(1 - \xi) \sim (M_W/M_Y)^4 \quad (4)$$

I deduce that if $(1 - \xi)$ could be measured to an accuracy of 10^{-4} the limit would become $M_Y \geq 10M_W$ and if to an accuracy 10^{-8} it would be $M_Y \geq 100M_W$. The first of these is within the realm of feasibility and certainly seems an important experiment to pursue. The group at the Paul Scherrer Institute near Zurich (Gerber, Fetscher) is one that is planning this experiment.

Neutrinos in the 331 Model

In the minimal 331 Model as described so far, neutrinos are massless and the model respects lepton number $\Delta L = 0$. Now I shall discuss soft L breaking for $M(\nu_i) \neq 0$.

Spontaneous breaking of L would lead to a massless (triplet) majoron in disagreement with experiment. Therefore I consider soft explicit breaking of L . The lepton families can be written $L_{i\alpha} = (l_i^-, \nu_i, l_i^+)_L$. Among the Higgs scalars are the $H^{\alpha\beta}$ sextet and the ϕ^α triplet. The Yukawa couplings are:

$$h_1^{ij} L_\alpha^i L_\beta^j H^{\alpha\beta} + h_2^{ij} L_\alpha^i L_\beta^j \bar{\phi}_\gamma^{\alpha\beta\gamma} + h.c. \quad (5)$$

The soft breaking of L is in the triple Higgs couplings:

$$m_1 H^{\alpha_1\beta_1} H^{\alpha_2\beta_2} H^{\alpha_3\beta_3} \epsilon_{\alpha_1\beta_1\gamma_1} \epsilon_{\alpha_2\beta_2\gamma_2} + m_2 (H^{\alpha\beta} \bar{\phi}_\alpha \bar{\phi}_\beta + h.c.) \quad (6)$$

The neutrinos acquire mass from one-loop insertions of the soft breaking and one finds that provided the VEV $\langle H^{22} \rangle = 0$ then there is the so-called cubic see-saw formula:

$$M(\nu_i) = CM(l_i^-)^3/M_W^2 \quad (7)$$

where l_i^- is the charged lepton corresponding to ν_i and C is a constant calculable in terms of various Yukawa couplings and Higgs masses but whose absolute value is redundant in the sequel. As an example, suppose I adopt the value for ν_τ of $29.3eV$, an impressively precise value predicted by Sciama's cosmology - obviously this is only an example! - then the other neutrinos have values $6.2meV$ and $690peV$ (where m is milli- and p is pico-).

The L breaking will also contribute to neutrinoless double beta decay but the rate is around a billion (10^9) times below present experimental limits.

The cubic see-saw with the cube of the charged lepton mass is numerically quite similar to the more familiar quadratic see-saw with the up quark mass, but since our present derivation does not involve a right-handed neutrino its origin is conceptually quite independent. In any case, I can fit the Hot Dark Matter and MSW requirements but not that for the atmospheric neutrinos simultaneously just as for the Gell-Mann et al and Yanagida case.

Phenomenology of the 331 Model

The dilepton can be produced in a hadron collider such as a pp or $p\bar{p}$ machine, or in a lepton collider such as e^+e^- or e^-e^- .

For the hadron collider the Y may be either pair produced or produced in association with an exotic quark [the latter carries $L = \pm 2$]. It turns out that the associated production is about one order of magnitude larger. These cross-sections are calculated in the literature - for a pp collider of the type envisioned there would be at least 10^4 striking events per year.

Surely the most dramatic way to spot a dilepton, however, would be to run a linear collider in the e^-e^- mode and find a direct-channel resonance. A narrow spike at between $300GeV$ and $800GeV$ would have a width at most a few percent of its mass and its decay to $\mu^-\mu^-$ has no standard model background.

Key Points of 331 Model:

- (i) The family symmetry can attempt to explain the fermion hierarchy and why there are three families.
- (ii) In the 331 Model, the neutrino mass is either zero or proportional to the cube of the charged lepton mass, depending on whether or not one softly breaks L .
- (iii) The dilepton ($300GeV$ - $800GeV$) could produce a narrow resonance in the e^-e^- mode.

Finite Groups as Family Symmetries

As a new topic, let me turn to consideration of generic models of the type where the symmetry group is $SM \times G$ with SM the standard group and G is a finite group under

which the families transform under some non-trivial representation. This has already been studied for the abelian groups Z_N and for certain non-abelian cases S_3 and S_4 .

Before focusing in on specific groups, let me step back and look at all finite groups of order $g \leq 31$. [It is normal to stop at $g = 2^n - 1$ because $g = 2^n$ is always so rich in groups.]

There are altogether 93 inequivalent such groups: 48 are abelian and the remaining 45 non-abelian. Groups with $g \geq 32$ might well also be interesting but surely lower g is simpler.

From any good textbook on finite groups[2] one may find a tabulation of the number of finite groups as a function of the order g , the number of elements in the group.

Amongst finite groups, the non-abelian examples have the advantage of non-singlet irreducible representations which can be used to inter-relate families. Which such group to select is based on simplicity: the minimum order and most economical use of representations[3].

Let me first dispense with the abelian groups. These are all made up from the basic unit Z_p , the order p group formed from the p^{th} roots of unity. It is important to note that the product $Z_p Z_q$ is identical to Z_{pq} if and only if p and q have no common prime factor.

If I write the prime factorization of g as:

$$g = \prod_i p_i^{k_i} \quad (8)$$

where the product is over primes, it follows that the number $N_a(g)$ of inequivalent abelian groups of order g is given by:

$$N_a(g) = \prod_{k_i} P(k_i) \quad (9)$$

where $P(x)$ is the number of unordered partitions of x . For example, for order $g = 144 = 2^4 3^2$ the value would be $N_a(144) = P(4)P(2) = 5 \times 2 = 10$. For $g \leq 31$ it is simple to evaluate $N_a(g)$ by inspection. $N_a(g) = 1$ unless g contains a nontrivial power ($k_i \geq 2$) of a prime. These exceptions are: $N_a(g = 4, 9, 12, 18, 20, 25, 28) = 2$; $N_a(8, 24, 27) = 3$; and $N_a(16) = 5$. This confirms that:

$$\sum_{g=1}^{31} N_a(g) = 48 \quad (10)$$

I shall not consider these abelian cases further, because all their irreducible representations are one-dimensional.

Of the nonabelian finite groups, the best known are perhaps the permutation groups S_N (with $N \geq 3$) of order $N!$. The smallest non-abelian finite group is S_3 ($\equiv D_3$), the symmetry of an equilateral triangle with respect to all rotations in a three dimensional sense. This group initiates two infinite series, the S_N and the D_N . Both have elementary geometrical significance since the symmetric permutation group S_N is the symmetry of the N -plex in N dimensions while the dihedral group D_N is the symmetry of the planar N -agon in 3 dimensions. As a family symmetry, the S_N series becomes uninteresting rapidly as the order and the dimensions of the representations

increase. Only S_3 and S_4 are of any interest as symmetries associated with the particle spectrum[5], also the order (number of elements) of the S_N groups grow factorially ($N!$) with N . The order of the dihedral groups increase only linearly ($2N$) with N and their irreducible representations are all one- and two- dimensional. This is reminiscent of the representations of the electroweak $SU(2)_L$ used in Nature.

Each D_N is a subgroup of $O(3)$ and has a counterpart double dihedral group Q_{2N} , of order $4N$, which is a subgroup of the double covering $SU(2)$ of $O(3)$.

With only the use of D_N , Q_{2N} , S_N and the tetrahedral group T (of order 12, the even permutations subgroup of S_4) I find 32 of the 45 nonabelian groups up to order 31, either as simple groups or as products of simple nonabelian groups with abelian groups: (Note that $D_6 \simeq Z_2 \times D_3$, $D_{10} \simeq Z_2 \times D_5$ and $D_{14} \simeq Z_2 \times D_7$)

| | |
|----|--|
| g | |
| 6 | $D_3 \equiv S_3$ |
| 8 | $D_4, Q = Q_4$ |
| 10 | D_5 |
| 12 | D_6, Q_6, T |
| 14 | D_7 |
| 16 | $D_8, Q_8, Z_2 \times D_4, Z_2 \times Q$ |
| 18 | $D_9, Z_3 \times D_3$ |
| 20 | D_{10}, Q_{10} |
| 22 | D_{11} |
| 24 | $D_{12}, Q_{12}, Z_2 \times D_6, Z_2 \times Q_6, Z_2 \times T,$ $Z_3 \times D_4, D_3 \times Q, Z_4 \times D_3, S_4$ |
| 26 | D_{13} |
| 28 | D_{14}, Q_{14} |
| 30 | $D_{15}, D_5 \times Z_3, D_3 \times Z_5$ |

There remain thirteen others formed by twisted products of abelian factors. Only certain such twistings are permissable, namely (completing all $g \leq 31$)

| | |
|----|---|
| g | |
| 16 | $Z_2 \tilde{\times} Z_8$ (two, excluding D_8), $Z_4 \tilde{\times} Z_4, Z_2 \tilde{\times} (Z_2 \times Z_4)$ (two) |
| 18 | $Z_2 \tilde{\times} (Z_3 \times Z_3)$ |
| 20 | $Z_4 \tilde{\times} Z_7$ |
| 21 | $Z_3 \tilde{\times} Z_7$ |
| 24 | $Z_3 \tilde{\times} Q, Z_3 \tilde{\times} Z_8, Z_3 \tilde{\times} D_4$ |
| 27 | $Z_9 \tilde{\times} Z_3, Z_3 \tilde{\times} (Z_3 \times Z_3)$ |

It can be shown that these thirteen exhaust the classification of *all* inequivalent finite groups up to order thirty-one[2].

Of the 45 nonabelian groups, the dihedrals (D_N) and double dihedrals (Q_{2N}), of order $2N$ and $4N$ respectively, form the simplest sequences. In particular, they fall into subgroups of $O(3)$ and $SU(2)$ respectively, the two simplest nonabelian continuous groups.

For the D_N and Q_{2N} , the multiplication tables, as derivable from the character tables, are simple to express in general. D_N , for odd N , has two singlet representations $1, 1'$ and $m = (N - 1)/2$ doublets $2_{(j)}$ ($1 \leq j \leq m$). The multiplication rules are:

$$1' \times 1' = 1; \quad 1' \times 2_{(j)} = 2_{(j)} \quad (11)$$

$$2_{(i)} \times 2_{(j)} = \delta_{ij}(1 + 1') + 2_{(\min[i+j, N-i-j])} + (1 - \delta_{ij})2_{(|i-j|)} \quad (12)$$

For even N , D_N has four singlets $1, 1', 1'', 1'''$ and $(m-1)$ doublets $2_{(j)}$ ($1 \leq j \leq m-1$) where $m = N/2$ with multiplication rules:

$$1' \times 1' = 1'' \times 1'' = 1''' \times 1''' = 1 \quad (13)$$

$$1' \times 1'' = 1'''; 1'' \times 1''' = 1'; 1''' \times 1' = 1'' \quad (14)$$

$$1' \times 2_{(j)} = 2_{(j)} \quad (15)$$

$$1'' \times 2_{(j)} = 1''' \times 2_{(j)} = 2_{(m-j)} \quad (16)$$

$$2_{(j)} \times 2_{(k)} = 2_{|j-k|} + 2_{(\min[j+k, N-j-k])} \quad (17)$$

(if $k \neq j, (m-j)$)

$$2_{(j)} \times 2_{(j)} = 2_{(\min[2j, N-2j])} + 1 + 1' \quad (18)$$

(if $j \neq m/2$)

$$2_{(j)} \times 2_{(m-j)} = 2_{|m-2j|} + 1'' + 1''' \quad (19)$$

(if $j \neq m/2$)

$$2_{m/2} \times 2_{m/2} = 1 + 1' + 1'' + 1''' \quad (20)$$

This last is possible only if m is even and hence if N is divisible by *four*.

For Q_{2N} , there are four singlets $1, 1', 1'', 1'''$ and $(N-1)$ doublets $2_{(j)}$ ($1 \leq j \leq (N-1)$).

The singlets have the multiplication rules:

$$1 \times 1 = 1' \times 1' = 1 \quad (21)$$

$$1'' \times 1'' = 1''' \times 1''' = 1' \quad (22)$$

$$1' \times 1'' = 1'''; 1''' \times 1' = 1'' \quad (23)$$

for $N = (2k+1)$ but are identical to those for D_N when $N = 2k$.

The products involving the $2_{(j)}$ are identical to those given for D_N (N even) above.

This completes the multiplication rules for 19 of the 45 groups. When needed, rules for the other groups will be derived.

Since I shall be emphasizing the groups Q_{2n} , let me be more explicit concerning it and its distinction from D_{2n} .

Q_{2n} is defined by the equations:

$$A^{2n} = E \quad (24)$$

$$B^2 = A^n \quad (25)$$

$$ABA = B \quad (26)$$

I can find an explicit matrix representation in the form:

$$A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \quad (27)$$

with $\theta = \pi/n$. This gives then:

$$A^n = \begin{pmatrix} \cos(n\theta) & \sin(n\theta) \\ -\sin(n\theta) & \cos(n\theta) \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad (28)$$

B is chosen as:

$$B = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \quad (29)$$

For D_{2N} , on the other hand the choice of B is replaced by:

$$B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (30)$$

so that $B^2 = A^n = +1$ instead of -1 .

From these matrices one can deduce, for example, the geometrical interpretation that whereas D_6 is the full dihedral [*i.e.* two-sided] symmetry of a planar hexagon in $O(3)$, Q_6 is the full $SU(2)$ symmetry of an equilateral triangle when rotation by 2π gives a sign (-1) and a rotation by 4π is the identity transformation.

Anomalies and Model Building.

The models I shall consider have a symmetry comprised of the standard model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ producted with a nonabelian finite group G.

If G is a global (ungauged) symmetry, there are problems if the spacetime manifold is topologically nontrivial since it has been shown that any such global symmetry is broken in the presence of wormholes[6]. From a Local viewpoint (Local with a capital means within a flat spacetime neighbourhood) the distinction between a global and local (gauged) finite symmetry does not exist. The distinction exists only in a Global sense (Global meaning pertaining to topological aspects of the manifold). In a flat spacetime, gauging a finite group has no meaning. In the presence of wormholes, expected from the fluctuations occurring in quantum gravity, gauging G is essential. The mathematical treatment of such a gauged finite group has a long history[7].

In order to gauge the finite group G, the simplest procedure is to gauge a continuous group H which contains G as a subgroup, and then to spontaneously break H by choice of a Higgs potential. The symmetry breaking may occur at a high energy scale, and then the low energy effective theory will not contain any gauge potentials or gauge bosons; this effective theory is, as explained above, Locally identical to a globally-invariant theory with symmetry G.

For example, consider $G = Q_6$ and $H = SU(2)$. I would like to use only one irreducible representation Φ of Q_6 in the symmetry-breaking potential $V(\Phi)$. The irreps. of Q_6 are $1, 1', 1'', 1''', 2, 2_S$. The $1'', 1'''$ and 2_S are spinorial and appear in the decompositions only of $2, 4, 6, 8, \dots$ of $SU(2)$. Since Φ must contain the 1 of Q_6 I must choose from the vectorial irreps. $3, 5, 7, 9, \dots$ of $SU(2)$. The appropriate choice is the 7 represented by a symmetric traceless third-rank tensor Φ_{ijk} with $\Phi_{ikk} = 0$.

For the vacuum expectation value, I choose

$$\langle \Phi_{111} \rangle = +1; \langle \Phi_{122} \rangle = -1 \quad (31)$$

and all other unrelated components vanishing. If I look for the 3×3 matrices R_{ij} which leave invariant this VEV I find from choices of indices in

$$R_{il}R_{jm}R_{kn} \langle \Phi_{lmn} \rangle = \langle \Phi_{ijk} \rangle \quad (32)$$

that $R_{31} = R_{32} = 0$ (Use $\langle \Phi_{3ij}\Phi_{3ij} \rangle = 0$) and that $R_{33} = \pm 1$. Then I find $(R_{11})^3 - 3R_{11}(R_{12})^2 = 1$ (Use $l = m = n = 1$ in (32)). This means that if $R_{11} = \cos \theta$ then $\cos 3\theta = 1$ or $\theta_n = 2\pi n/3$. So the elements of Q_6 are $A = R_3(\theta_1), A^2, A^3$ and B, BA, BA^2 where $B = \text{diag}(i, -i, -i)$.

More generally, it can be shown that to obtain Q_{2N} one must use an N^{th} rank tensor because one finds for the elements R_{11} and R_{12} :

$$\sum_{p=0}^{[N/2]} (-1)^p \binom{N}{2p} (R_{11})^{N-2p} (R_{12})^{2p} = \cos N\theta = 1 \quad (33)$$

If the group H is gauged, it must be free from anomalies. This entails several conditions which must be met:

(a) The chiral fermions must fall into complete irreducible representations not only of G but also of H .

(b) These representations must be free of all H anomalies including $(H)^3, (H)^2Y$; for the cases of $H = O(3), SU(2)$ only the latter anomaly is nontrivial.

(c) If $H = SU(2)$, there must be no global anomaly.

The above three conditions apply to nonabelian H . The case of an abelian H avoids (a) and (c) but gives rise to additional mixed anomalies in (b).

For nonabelian H , conditions (b) and (c) are straightforward to write down and solve. Condition (a) needs more discussion. I shall focus on the special cases of $O(3) \supset D_N$ and $SU(2) \supset Q_{2N}$.

For $O(3)$ the irreps. are **1, 3, 5, 7,** dimensional. D_N has irreps. (for even $N = 2m$) $1, 1', 1'', 1'''$ and $2_{(j)} (1 \leq j \leq (m-1))$ and these correspond to:

$$O(3) : \mathbf{1} \rightarrow 1; \mathbf{3} \rightarrow (1' + 2_{(1)}) \quad (34)$$

and so on. The same situation occurs for odd N with irreps. $1, 1'$ and $2_{(j)} (1 \leq j \leq (N-1)/2)$.

For $SU(2) \supset Q_{2N}$ the corresponding breakdown is:

$$\mathbf{1} \rightarrow 1; \mathbf{2} \rightarrow 2_{S(1)}; \mathbf{3} \rightarrow 1' + 2_{(1)} \quad (35)$$

and so on, where the doublets of Q_{2N} , $2_{(1)}$ and $2_{S(1)}$, are defined by Eq. (35).

These are the principal splittings of a continuous group irrep. into finite subgroup irreps. I shall need in my discussions of model building.

Applications of Q_{2N}

There has recently been considerable interest in the structure of the quark mass matrices, particularly in the idea of postulating texture zeros in grand unified theories,

with a view to obtaining relations between the masses and mixing angles[8], [9], [10]. A list of phenomenologically viable quark mass matrices bearing a maximum number of symmetric texture zeros was presented in [11]. The possibility of constructing such mass matrices from a scheme of gauged flavor symmetry has been considered in [12] and [13].

There has also been considerable activity in the use of finite non-abelian groups as flavor symmetries[14],[15],[16], with a view to generating the mass hierarchy.

Here I attempt a synthesis of these two approaches and construct desirable quark mass matrix textures by using a nonabelian flavor symmetry (specifically a dicyclic group Q_{2N}) together with the Froggatt-Nielsen mechanism of mass generation[17].

The particular choices are given in [21]. A generalization (and simplification) to include supersymmetric grand unification is given in [22,23].

While $U(1)$ flavor symmetry constructions for quark mass matrices with *nonsymmetric* hierarchical textures have been attempted [18], the full list of such phenomenologically viable quark mass matrices is not yet available. My approach does not *a priori* give a symmetric texture. However, as a first attempt, I consider here only the possibility of constructing the symmetric texture patterns presented in [11]. The general case of non-symmetric textures would naturally be a very interesting next step.

The use of Q_{2N} as a finite flavor group has been discussed in detail in [16]. I recall here that the irreducible representations of Q_{2N} are four singlets $1, 1', 1'', 1'''$ and $(N-1)$ doublets $2_{(k)}$, with $1 \leq k \leq (N-1)$. Most important for my purposes are the products:

$$2_{(k)} \times 2_{(l)} = 2_{(|k-l|)} + 2_{(\min\{k+l, 2N-k-l\})} \quad (36)$$

where, in a generalized notation, $2_{(0)} \equiv 1 + 1'$ and $2_{(N)} \equiv 1'' + 1'''$.

I assign the quarks to Q_{2N} representations as follows:

$$\begin{array}{ccc} \left\{ \begin{pmatrix} t \\ b \\ c \\ s \end{pmatrix}_L \right\} & 2_{(2)} & \left\{ \begin{matrix} t_R \\ c_R \\ b_R \\ s_R \end{matrix} \right\} & \begin{matrix} 2_{(2)} \\ 2_{(1)} \end{matrix} \\ \left(\begin{pmatrix} u \\ d \end{pmatrix}_L \right) & 1' & \begin{matrix} u_R \\ d_R \end{matrix} & \begin{matrix} 1' \\ 1 \end{matrix} \end{array}$$

When I embed the finite spinorial Q_{2N} into its continuous progenitor $SU(2)$, $1, 2_{(1)}$ and $(1' + 2_{(2)})$ correspond respectively to the singlet, doublet and triplet representations. The above quark assignment is thus anomaly-free if the leptons are assigned to:

$$\begin{array}{ccc} \left\{ \begin{pmatrix} \nu_\tau \\ \tau^- \\ \nu_\mu \\ \mu^- \end{pmatrix}_L \right\} & 2_{(1)} & \left\{ \begin{matrix} \tau_L^+ \\ \mu_L^+ \end{matrix} \right\} & 2_{(2)} \\ \left(\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \right) & 1 & e_L^+ & 1' \end{array}$$

I shall not consider lepton masses further here. For the mass textures of the quarks I postulate heavy vector-like fermions and singlet Higgs and assume the quark masses arise from tree graphs as in [17].

As the first of two successful examples, I demonstrate how to derive the five-zero texture in Eqs. (37) and (38) below. [Note that no texture with the maximum number of six texture zeros can be phenomenologically viable.]

$$M_u = \begin{pmatrix} 0 & 0 & \lambda^4 \\ 0 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \quad (37)$$

$$M_d = \begin{pmatrix} 0 & \lambda^4 & 0 \\ \lambda^4 & \lambda^3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (38)$$

For these matrices I have suppressed all coefficients of order one since at the present stage I am satisfied to derive only the correct orders in λ for each entry.

The standard Higgs scalar doublets of $SU(2)_L$ are taken as a $2_{(4)}$ of Q_{2N} coupling to the up quarks, and a $2_{(3)}$ coupling to the down quarks. I assume these get VEVs that break Q_{2N} and give mass only to the third family. For the up quark mass matrix the entry $(M_u)_{33}$ is of order 1 from the coupling $t_L(2_{(2)})t_R(2_{(2)})H_u(2_{(4)})$. At leading order, all other entries $(M_u)_{ij}$ vanish. Similarly, $b_L(2_{(2)})b_R(2_{(1)})H_d(2_{(3)})$ gives $(M_d)_{33}$ of order 1 and no other $(M_d)_{ij}$.

To obtain the other entries in Eqs. (37) and (38) at order λ^n ($\lambda \sim \sin\theta_C \sim 0.22$ where θ_C is the Cabibbo angle), I introduce a list of vector-like quark doublets $Q_i(2_{(i)})(i = 6, 7, 10, 13, 14)$, singlets $U_i(2_{(i)})(i = 6, 10, 14)$ and $D_i(2_{(i)})(i = 4, 17)$, bearing the same standard model quantum numbers as Q_L, u_R and d_R respectively, together with standard model singlet Higgses $S_i(2_{(i)}), (i = 5, 8, 13, 14, 17, 20)$. Although this set of Q_{2N} doublets seems long and *ad hoc*, it is highly constrained (see below). Since I have assumed heavy particles in doublets up to 2_{20} the flavor group, of order 84, is Q_{42} .

I choose a set of bases and label the two states in the heavy fermion Q_{2N} doublets as $2_{(i)+}$ and $2_{(i)-}$, which lie respectively in the third and second family direction. The H_u VEV then allows only the six couplings:

$$t_L < H_u > U_{6+}; \quad Q_{6+} < H_u > t_R; \quad Q_{6+} < H_u > U_{10+};$$

$$Q_{10+} < H_u > U_{6+}; \quad Q_{10+} < H_u > U_{14+}; \quad Q_{14+} < H_u > U_{10+}.$$

and the H_d VEV only the two couplings:

$$Q_{7+} < H_d > D_{4+}; \quad Q_{14+} < H_d > D_{17+}.$$

The S_i VEVs may then be chosen to give certain vertices such as : $U_{6+}^\dagger < S_{8+-} > c_R$, $U_{10+}^\dagger < S_{8+-} > c_R$, and others. I define:

$$\lambda^2 = \frac{\langle S_i \rangle}{M_{even}} \quad (39)$$

$$\lambda = \frac{\langle S_i \rangle}{M_{odd}} \quad (40)$$

where M_{even} and M_{odd} denote the mass of a heavy fermion in Q_{2N} representation $2_{(k)}$ for being k even and odd respectively. Note that Eqs. (39) and (40) are acceptable

because the k-even and k-odd doublets occur independently in the irreducible representations of the covering $SU(2)$ in the sense that the k-even doublets appear only in vectors of $SU(2)$ and the k-odd doublets appear only in spinors.

I have now all the ingredients of the model. In the low energy effective field theory, after integrating out the heavy fermions[19], I have the tree level quark mass matrices having the structure of the model denoted by roman numeral V in ref. [11], namely those exhibited in Eqs.(37) and (38) above.

For instance, the $(M_u)_{32}$ entry is given by the Froggatt-Nielsen tree graph (shown in Fig.(1a)) corresponding to the operator couplings

$$t_L < H_u > (U_{6+} U_{6+}^\dagger) < S_{8+-} > c_R = \lambda^2 < H_u > t_L c_R;$$

while $(M_u)_{13}$ is given by the graph of Fig. (1c) corresponding to:

$$\begin{aligned} t_L < H_u > (U_{6+} U_{6+}^\dagger) < S_{8+-} > (U_{14-} U_{14-}^\dagger) < S_{14-} > u_R \\ = \lambda^4 < H_u > t_L u_R; \end{aligned}$$

and $(M_d)_{22}$ is given by Fig. (2a) corresponding to:

$$\begin{aligned} s_L < S_{5+-} > (Q_{7+}^\dagger Q_{7+}) < H_d > (D_{4+} D_{4+}^\dagger) < S_{5+-} > s_R \\ = \lambda^3 < H_d > s_L s_R. \end{aligned}$$

The other entries in M_u and M_d are derived similarly; some further examples are shown in Fig. (1b) for M_u and Figs. (2b) and (2c) for M_d .

In the construction of the model, I followed a systematic procedure and were surprised to realize that it is highly non-trivial if any consistent model can be constructed at all. The difficulty is not only to derive the correct texture zeros but also to avoid unwanted entries at too low an order in λ . I find only two consistent models, the above model and one alternative summarized below.

The mass matrices for the alternative model are:

$$M_u = \begin{pmatrix} 0 & \lambda^6 & 0 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix} \quad (41)$$

$$M_d = \begin{pmatrix} 0 & \lambda^4 & 0 \\ \lambda^4 & \lambda^3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (42)$$

Note that the M_d matrix is the same as in my first example but M_u is changed; as before, I neglect coefficients of order unity. The Q_{2N} assignments for the quarks and leptons are the same as they were previously.

The entries in Eqs. (41) and (42) at order λ^n are constructed, as in the previous example, through introducing vector-like Q_{2N} quark doublets $Q_i(2_{(i)})(i = 6, 7, 10, 18, 23)$, singlets $U_i(2_{(i)})(i = 6, 10, 18)$ and $D_i(2_{(i)})(i = 4, 11)$, together with standard model singlet Higgses $S_i(2_{(i)}), (i = 5, 8, 11, 15, 16, 18, 23)$. As mentioned in my first example this set of doublets which seems long and *ad hoc* is really highly constrained. For this second example, the flavor group is Q_{48} .

Under the same kind of state labelling, the H_u VEV then allows only the four couplings:

$$t_L < H_u > U_{6+}, \quad Q_{6+} < H_u > t_R, \\ Q_{6+} < H_u > U_{10+}, \quad Q_{10+} < H_u > U_{6+};$$

and the H_d VEV only the coupling:

$$Q_{7+} < H_d > D_{4+}.$$

The S_i VEVs may then be chosen to give certain vertices such as : $U_{6+}^\dagger < S_{8+-} > c_R$, $U_{10+}^\dagger < S_{8+-} > c_R$, and so on.

The general procedure is as follows: the quarks and leptons has to come from $SU(2)_H$ singlets, doublets and triplets. There are only 21 anomaly free schemes of assignment with no extra chiral fermions[20]. I am aiming at assignments that can lead to up- and down-quark mass matrices with different hierarchical textures[11]. That leaves me with two schemes of which only the one used here gives interesting models.

Picking the above scheme, the feasibility of using the Froggatt-Nielsen mechanism enforces the first family quarks to be Q_{2N} singlets. I then introduce appropriate heavy fermions and Higgses whenever necessary as I go on to build entries of higher order in λ , keeping track of overall consistency.

Attempts to construct models giving texture models I, II, and III of ref. [11] lead to conflicts, and I therefore conclude that those patterns of texture zeros are disfavored.

In this approach, the standard model gauge group $G = SU(3)_C \times SU(2)_L \times U(1)_Y$ is extended to $G \times (Q_{2N})_{global}$ which, in turn, is assumed a subgroup of $G \times SU(2)_H$ where $SU(2)_H$ is a gauged horizontal symmetry. This last point is important because the imposition of the necessary anomaly cancellation restricts the assignment of the quarks and leptons to Q_{2N} representations as discussed above.

The authors of [11] have analyzed all possible symmetric quark mass matrices with the maximal (six) and next-to-maximal (five) number of texture zeros, and concluded that only five models, denoted by the roman numerals I to V in their work, are phenomenologically viable. By insisting on derivation of the texture zeros from the Q_{2N} dicyclic flavor symmetry, I have reduced the number of candidates to two.

Similar considerations can be made at the $SU(5) \times SU(5) \times Q_{2N}$ level, and this - together with the generalization to supersymmetry - is discussed in [22] and [23].

In conclusion, the reduction in the number of free parameters in the low energy theory attained by postulating texture zeros in the fermion mass matrices has been shown to have a dual description in terms of a horizontal symmetry $Q_{2N} \subset SU(2)_H$. This $SU(2)_H$ could arise in a GUT group or directly from a superstring. My main point is that the derivation of the values of the fermion masses and quark mixings in a putative theory of everything may likely involve a horizontal symmetry, probably gauged, as an important intermediate step. The simple cases given in this talk illustrate how this can happen.

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$$SU(N) \rightarrow SU(5) \times SU(5 - N) \times U(1) \rightarrow SU(5) \times G \quad (43)$$

where G is a discrete group. Such models typically have more than just three complete families until G is broken.

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Figure Captions.

Fig.1 Froggatt-Nielsen tree graphs for M_u . (The symmetric counterpart $(M_u)_{23}$, and second graphs for $(M_u)_{22}$ and $(M_u)_{33}$ are not shown).

Fig.2 Froggatt-Nielsen tree graphs for M_d .

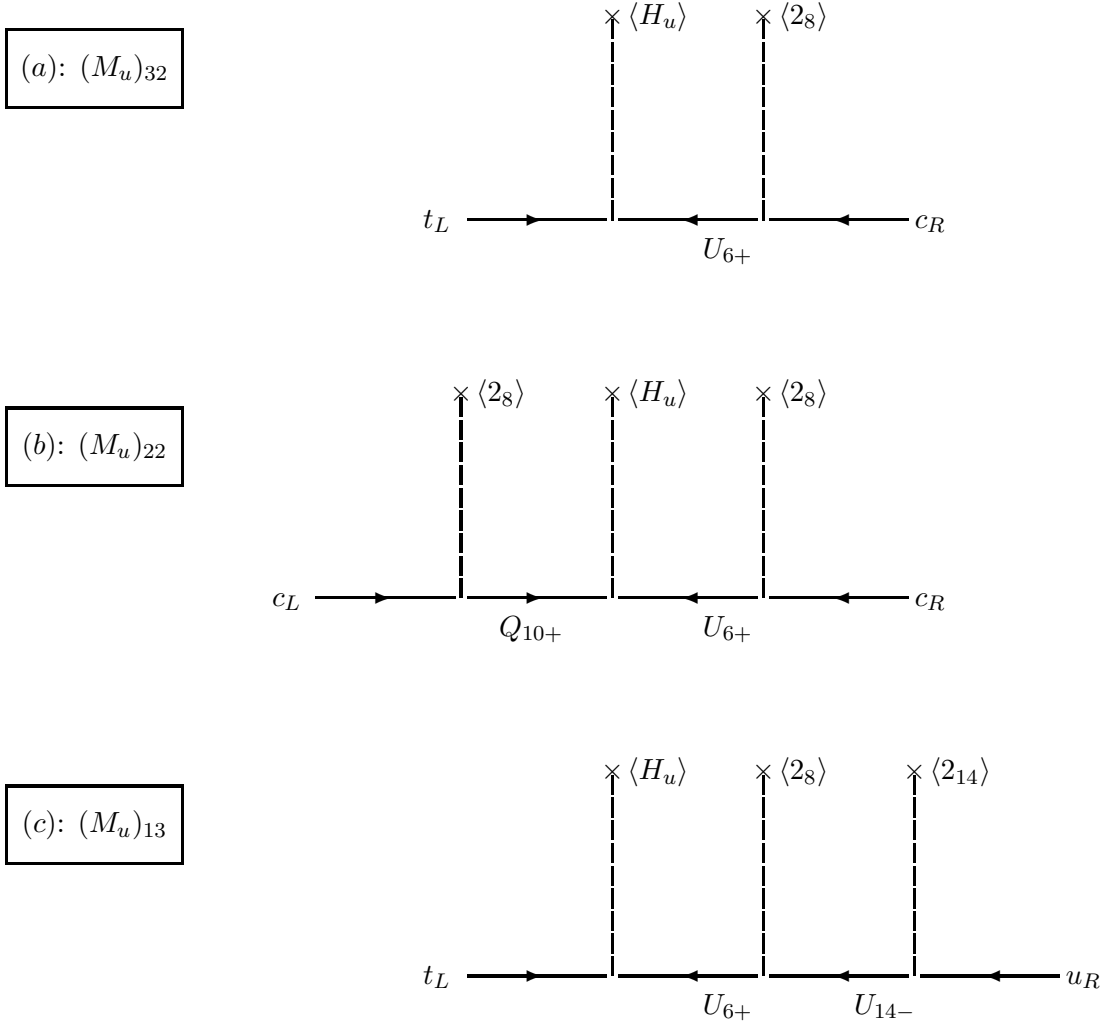
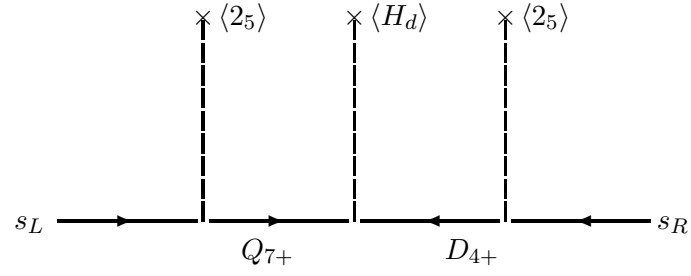
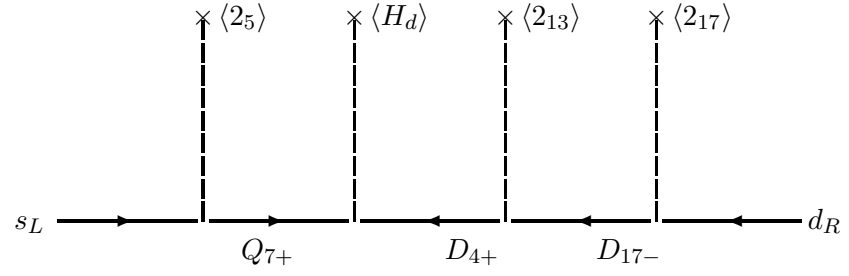


Figure 1: Froggatt-Nielsen tree graphs for M_u . (The symmetric counterpart $(M_u)_{23}$, and second graphs for $(M_u)_{22}$ and $(M_u)_{31}$ are not shown)

(a): $(M_d)_{22}$



(b): $(M_d)_{21}$



(c): $(M_d)_{12}$

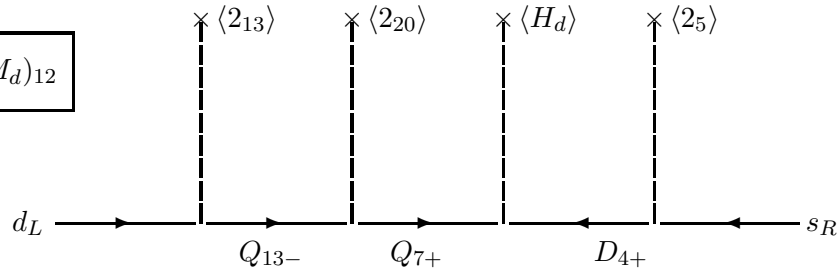


Figure 2: Froggatt-Nielsen tree graphs for M_d .